

8-14 立体図形の諸表

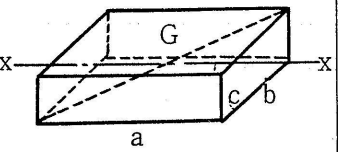
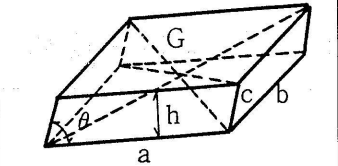
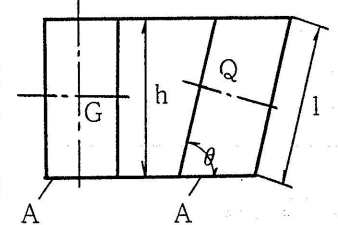
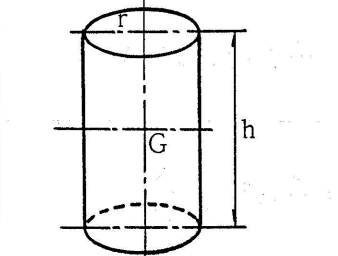
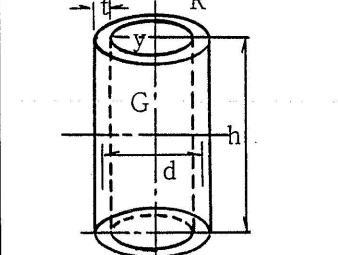
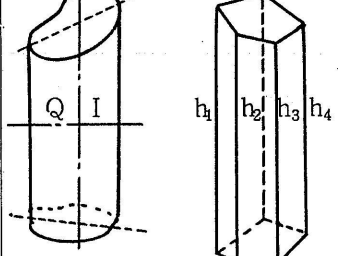
V : 立体の体積

M : とう体のとう面積又は錘体の錘面積

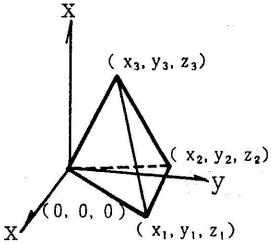
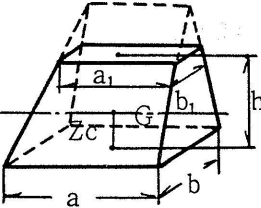
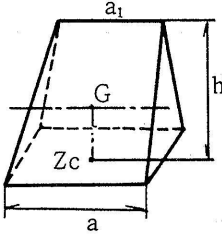
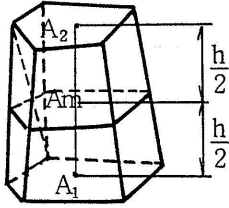
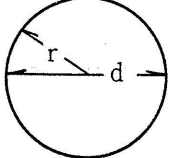
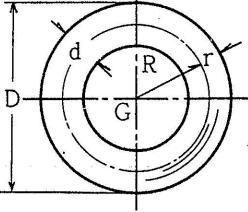
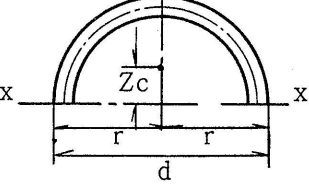
F : 立体の全表面積

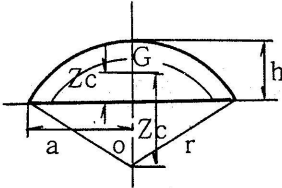
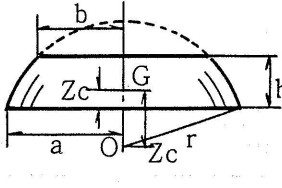
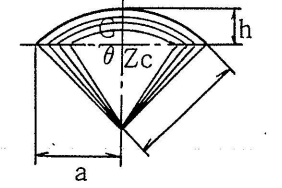
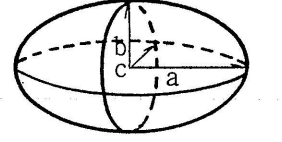
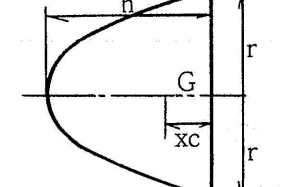
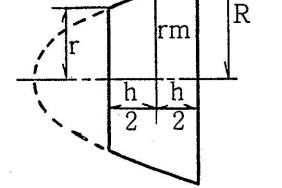
A : とう体又は錘体の底面積

G : 立体の重心

直六面体		$V = abc$ $F = 2(ab + bc + ca)$ $d = \sqrt{a^2 + b^2 + c^2} \text{ ただし } d \text{ は対角線の長さ}$
平行六面体		$V = abh = abc \sin \theta, \quad h = c \sin \theta$ $F = 2[ab + (a+b)h] = 2[ab + (a+b)c \sin \theta]$
とう体		$V = Ah = Ql$ $M = Uh = Cl$ $F = M + 2A$ $Q = A \sin \theta$ <p style="text-align: center;"> A = 底面積 C = 横断面の周辺長 Q = 横断面積 U = 底面の周辺長 </p>
直円塔		$V = \pi r^2 h$ $M = 2\pi r h$ $A = \pi r^2$ $F = 2\pi r(r + h)$
中空円塔		$V = \pi(R^2 - r^2)h = \pi(2R - t)th$ $= \pi(2r + t)th$ $= \pi dth$
斜載とう体		$V = Ql = \frac{Q}{n}(h_1 + h_2 + \dots + h_n)$ <p style="text-align: center;"> l = 両端面の図心を結ぶ線分の長さ h = 角とうにおける各辺の長さ Q = とう体の軸 l に垂直な横断面積 </p>

斜載直円とう		$V = \pi r^2 h = \frac{1}{2} \pi r^2 (h_1 + h_2)$ $M = 2\pi r h = \pi r (h_1 + h_2)$ $y_c = \frac{r^2 \tan \theta}{4h}, \quad Z_c = \frac{h}{2} + \frac{r^2 \tan^2 \theta}{8h}$
蹄形		$V = \frac{h}{3b} [2a^2 - 3(r-b)r^2\phi + 3(r-b)^2a]$ $M = \frac{2rh}{6} [a + (b-r)\phi]$ <p>底面が半円の場合 ($a=b=r$)</p> $V = \frac{2r^2h}{3}, \quad M = 2rh, \quad x_c = \frac{3}{16}\pi r, \quad Z_c = \frac{3}{32}\pi h$
載頭スイ体		$V = \frac{h}{3} [A + \sqrt{AB} + B]$ $Z_c = \frac{h}{4} \frac{A + 2\sqrt{AB} + 3B}{A + \sqrt{AB} + B}$ <p>A, B = 載頭スイ体の底面積 とくに角スイにおいては</p> $V = \frac{Ah}{3} \left[1 + \frac{b}{a} + \left(\frac{b}{a} \right)^2 \right]$
直円スイ		$V = \frac{\pi r^2 h}{3}, \quad M = \pi r \sqrt{r^2 + h^2} = \pi r s$ $s = \sqrt{r^2 + h^2}$ $F = M + A = \pi r (\sqrt{r^2 + h^2} + r) = \pi r (s + r)$
載頭円スイ		$V = \frac{\pi h}{3} (R^2 + Rr + r^2) = \frac{\pi h}{4} \left(a^2 + \frac{1}{3} b^2 \right)$ $M = \pi a s \quad a = R + r$ $Z_c = \frac{h}{4} \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}$
直四角スイ		$V = \frac{abh}{3}$
多角スイ		$V = \frac{Bh}{3} \quad B = \text{底面積}$ $Z_c = \frac{h}{4}$

三角 ス イ		$V = \pm \frac{1}{6} \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ <p>ただし4頂点の座標をそれぞれ(0, 0, 0), (x₁, y₁, z₁), (x₂, y₂, z₂), (x₃, y₃, z₃)とする。</p>
オ ペ リ ス ク		$V = \frac{h}{6} [(2a+a_1)b + (2a_1+a)b_1]$ $= \frac{h}{6} [ab + (a+a_1)(b+b_1) + a_1b_1]$ $Z_c = \frac{h}{2} \frac{ab + ab_1 + a_1b + 3a_1b_1}{2ab + ab_1 + a_1b + 2a_1b_1}$
く さ び 形		$V = \frac{bh}{6} (2a+a_1)$ $Z_c = \frac{h}{2} \frac{a+a_1}{2a+a_1}$
プ リ ズ モ イ ド		$V = \frac{h}{6} (A_1 + 4A_m + A_2)$ <p>A₁, A₂: 平行な両端面積 h: A₁, A₂の垂直距離 A_m: $\frac{h}{2}$の断面積</p>
球		$V = \frac{4}{3} \pi r^3 = 4.189r^3 = \frac{\pi}{6} d^3 = 0.524d^3$ $F = 4\pi r^2 = \pi d^2$
円 形 断 面 の 環		$V = 2\pi^2 Rr^2 = 19.739Rr^2 \approx 20Rr^2$ $F = 4\pi^2 Rr = 39.478Rr \approx 40Rr$
半 球		$V = \frac{2}{3} \pi r^3 = \frac{\pi}{12} d^3$ $M = 2\pi r^2 = \frac{\pi d^2}{2}$ $Z_c = \frac{3}{8} r = 0.375r$

欠球		$V = \frac{\pi h}{2} (3a^2 + h^2) = \frac{\pi h^2}{3} (3r - h)$ $M = \pi(a^2 + h^2) = 2\pi r h$ $Z_c = \frac{h}{4} \frac{4r - h}{3r - h}, \quad Z_c = \frac{3}{4} \frac{(2r - h)^2}{3r - h}$ $a^2 = h(2r - h)$
球帯		$V = \frac{\pi h}{6} (3a^2 + 3b^2 + h^2), \quad M = 2\pi r h$ $Z_c = \frac{h}{2} \frac{2a^2 + 4b^2 + h^2}{3a^2 + 3b^2 + h^2}$ $Z_c' = \frac{3}{2} \frac{a^4 - b^4}{(3a^2 + 3b^2 + h^2)h}$ $r^2 = a^2 + \left(\frac{a^2 - b^2 - h^2}{2h} \right)^2$
球底円錐		$V = \frac{2}{3} \pi r^2 h, \quad F = \pi r(2h + a)$ $Z_c = \frac{3r}{8} (1 + \cos \theta) = \frac{3}{8} (2r - h)$ $h = r(1 - \cos \alpha), \quad a = r \sin \alpha$
橢圓体		$V = \frac{4}{3} \pi abc$ <p>回転橢圓体の場合</p> $b = c$ $V = \frac{4}{3} \pi ab^2$
回転放物線体		$V = \frac{\pi}{2} r^2 h$ $x_c = \frac{h}{3}$
回転放物線体の截片		$V = \frac{\pi}{2} (R^2 + r^2) h = \pi r m^2 h$